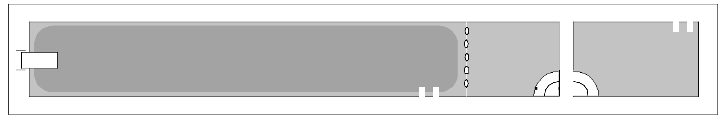
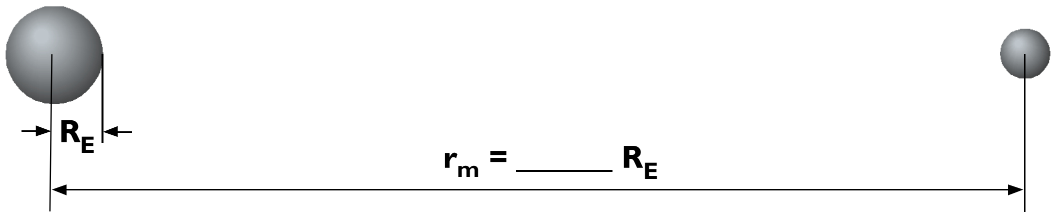
PhyzSpringboard:
The Apple and the Moon



FROM THE EARTH TO THE MOON

1. What is the distance to the moon in terms of the radius of the earth?



2. An apple falls about 193 inches (~16 ft) in one second near the surface of the earth. According to Newton's theory of universal gravitation, how far would an apple fall if it were dropped from a height equal to the distance to the moon? Follow these steps to find out.

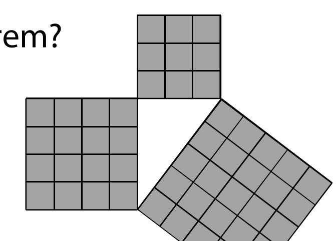
a. Simplify the ratio of gravitational acceleration at the moon's distance (a_m) to the gravitational acceleration near the earth's surface (a_E), using the distance to the moon described in part 1.

b. The distance an object falls in one second is proportional to its acceleration. That is, $d_m/d_E = a_m/a_E$. Determine the distance an apple would fall if it were as far from the earth as the moon is. This is the distance **predicted** by Newton's theory of universal gravitation.

PYTHAGOREAN REVIEW

3. State the Pythagorean Theorem and the conditions under which it applies.

4. How does the diagram to the right represent the Pythagorean Theorem?



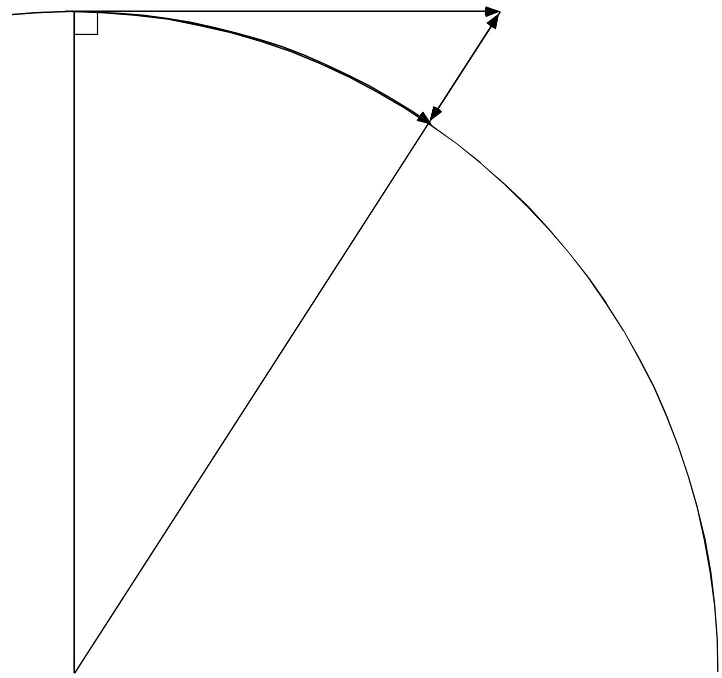
SMALL SQUARED

5. What is the result when you square a small quantity (a quantity less than 1)?

__A smaller quantity. __A larger quantity. __The same quantity you started with.

ORBITAL DETAILS

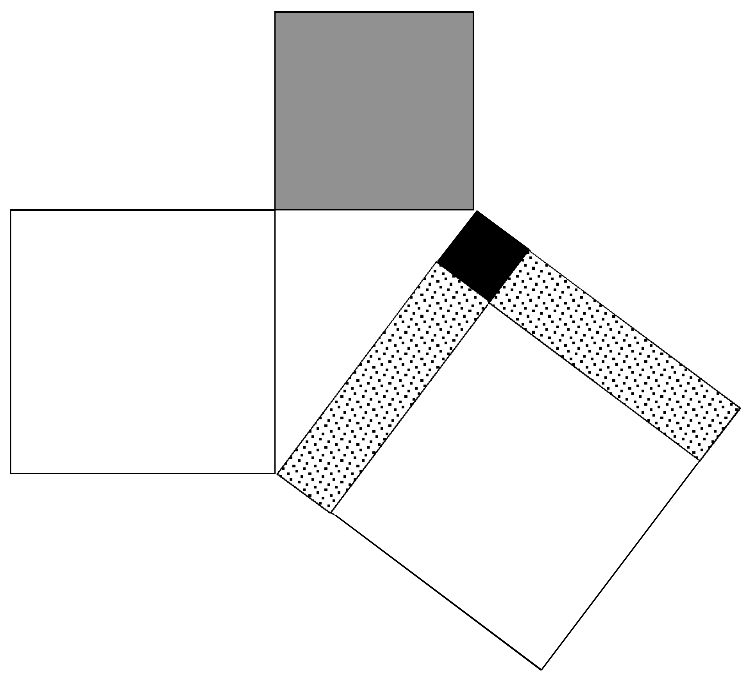
According to Newton's theory of universal gravitation, the moon **should** fall 1/20 of an inch in one second. What Newton needed to know to prove his theory was just how far the moon actually **does** fall in one second.



But the moon doesn't appear to fall at all. Not in one second, a day, a week, a month, or even a year. But the moon **does** fall. Not by getting closer to the center of the earth like an apple does. But rather by moving inward from the straight line path it would follow if there were no gravity.

Consider the portion of the moon's orbit shown to the right. Let *r* equal the earth-moon distance. Let *d* equal the distance the moon would travel in one second if there were no gravity. And let *s* equal the distance the moon falls away from that straight-line path in order to maintain a circular orbit.

6. a. Label all distances (*r*, *d*, and *s*) in the diagram above and the corresponding right triangle diagram to the right.



b. Identify each of the quantities in the diagram to the right. (Use only *r*, *d*, and *s*.)

i. White area:

ii. Gray area:

iii. Black area:

iv. Pebbled area (total):

7. Consider this statement of the Pythagorean Theorem for this triangle:

The white area plus the gray area equals the white area plus the pebbled area plus the black area.

a. Rewrite that statement using the symbols *r*, *s*, and *d*.

b. The black area is considered small enough to be ignored (small distance squared).
   i. What, then, can be concluded about the pebbled area and the gray area?

   ii. Write an expression using symbols.

8. Rearrange the expression in 7 to solve for *s*. (Remember that *s* is the distance we're looking for.)

9. The distance *d* that the moon would travel if there were no gravity and the length of the arc it actually travels are nearly equal to each other. So let *d* represent the arclength the moon would travel in one second along its circular orbit.

   d = arclength—distance traveled in t = one second

   $2\ \ r$ = circumference—the distance traveled in one ______________________

   t = one second—the time for the moon to travel a distance d

   T = one month—the time for the moon to travel a distance ______________________

10. Newton used ratios to determine how far the moon moved in one second. The ratio of the distance moved in one second to the distance moved in one month is equal to the ratio of one second to one month. That is, $d/2\ \ r = t/T$. He solved for *d* and used the expression in part 8 to find *s*. Retrace his steps.
a. What is the orbital radius (*r*) of the moon in inches? (The radius of the moon's orbit is 240,000 mi. There are 5280 ft in one mile and 12 inches in each foot.)

b. What is the distance the moon moves in one month (i. e., the length of the orbit, the circumference of the circle it sweeps out)? Hint: $2\ \ r$.

c. How long is one month in seconds? (One month is 27.3 days, one day is 24 hours, and one hour is 3600 seconds.)

d. How many inches does the moon move along its orbital path in one second? This is *d*. Hint: $2\ \ r/T$.

e. Use this value of *d* and the expression in part 8 to determine how far (*s*) the moon falls in one second.

11. This value of *s* is the distance the moon *actually* falls in one second as it orbits the Earth. Why is this small distance considered excellent proof of Newton's theory of universal gravitation?