PhyzGuide: Simple Harmonic Motion

What do the following items have in common: a plucked guitar string, an orbiting planet, a pendulum, the air in an organ pipe, and a building during an earthquake? They are but a few examples of **periodic motion** that occur in the universe. An object in periodic motion undergoes a cycle of motion repeatedly.

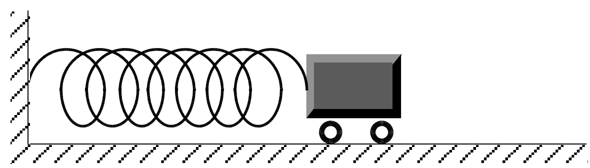
When considering orbits or rotational motion, we are dealing with **angular periodic motion**. Particles on a body in angular periodic motion typically trace out a circle; upon the completion of one circle, the cycle repeats itself (often on the same circle).

Vibrating guitar strings and columns of air in organ pipes are examples of **vibrational periodic motion**. Vibrational periodic motion is also referred to as **harmonic motion** (a reference arising from the association with music).

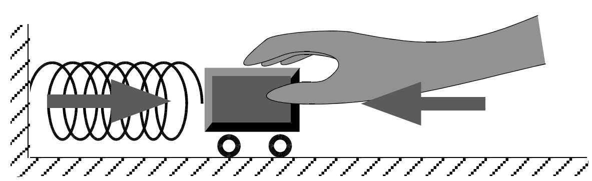
In keeping with our tradition of starting with simple models and adding layers of complexity as our level of understanding increases, we will begin with the study of simple harmonic motion. Real vibrating systems oscillate in complicated ways. Happily, these complicated vibrations are just several simple vibrations occurring simultaneously.

ENOUGH WITH THE PRELIMS AND CAVEATS; LET'S SHAKE IT!

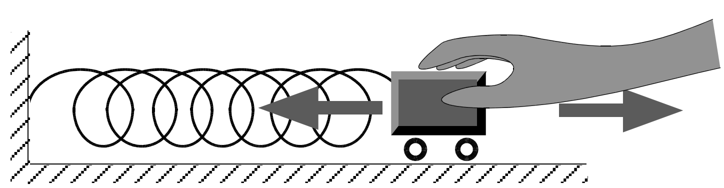
Simple harmonic motion (SHM) occurs when a system in stable equilibrium is "tweaked" (i.e. displaced slightly from equilibrium and released). Consider, for example, a system composed of a mass attached to a spring as shown to the right. When the system is at equilibrium, it is stable. If the mass is pushed to the left, the spring will push the mass to the right; if the mass is pulled to the right, the spring will pull the mass to the left. The force the spring exerts is referred to as the **restoring force** and follows Hooke's Law. It is called restoring force because it pushes or pulls the system back into equilibrium. We refer to the restoring force in this case as a **linear** restoring force because, by Hooke's Law ($F = kx$), the magnitude of this force is directly proportional to the distance by which the system is moved from equilibrium. Why all this attention to restoring force? *Any system that can undergo simple harmonic motion is governed by a linear restoring force*.



A mass attached to a spring. When the spring is neither compressed nor extended, the system is in equilibrium. No net force acts on the mass.

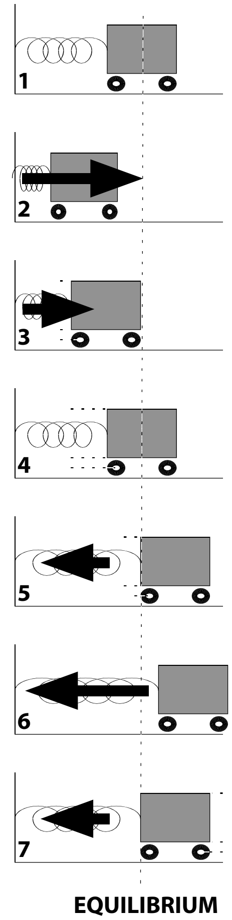


When the mass is pushed to the left, the spring pushes back with a force to the right.



When the mass is pulled to the right, the spring pulls back with a force to the left.

SHM: FORCE, POSITION, VELOCITY, AND ACCELERATION



Consider again our stable system (figure 1). Suppose the mass is pushed to the left and released (figure 2). The linear restoring force pushes the mass to the right, so the mass accelerates to the right with increasing rightward velocity. But as the mass approaches the equilibrium position (figure 3), the restoring force decreases (Hooke's Law). When the mass reaches the equilibrium position (figure 4), the restoring force is zero. Of course by this time, the mass has gained a certain velocity. Since there is no force acting on it, the mass continues past equilibrium. Once it is to the right of equilibrium (figure 5), the restoring force reappears—but this time, acting to the left. The restoring force now causes an acceleration to the left. This leftward force reduces the rightward motion of the mass. Eventually, the mass reaches a point as far to the right of equilibrium as it will go (figure 6). At this instant, the mass is at rest. But since the mass is still a distance to the right of equilibrium, the restoring force continues to pull to the left. And so the mass continues its leftward acceleration with an increasing leftward velocity (figure 7). It will again overshoot equilibrium and start the cycle over. This motion could theoretically cycle indefinitely. (Energy losses prevent this from happening in reality.)

The preceding description of a cycle of SHM focused on force, position, velocity, and acceleration. It is equally instructive to analyze the cycle in terms of momentum and energy (I smell a PhyzJob in the very near future).

SHM: PERIOD

The interval of time that passes during one cycle of SHM is called the period. It is denoted with a T and measured in seconds. There is an unexpected yet highly useful connection between vibrational periodic motion and angular vibrational motion. Through this connection, a simple expression for the period of a harmonic oscillator has been found.

$T = 2\pi\sqrt{(x/a)}$

where x is the distance the mass is from equilibrium and a is the acceleration of the mass. This equation is general; we will apply it to two special (and important) cases below.

PERIOD OF A SPRING-MASS OSCILLATOR

Applying this relation to a spring-mass oscillator whose force constant is k and whose mass is m (like the one we've been considering throughout this PhyzGuide), we find the following. Whenever the mass is at a distance x from equilibrium, the restoring force is kx. Its acceleration is therefore kx/m. So

$T = 2\pi\sqrt{(x/a)} = 2\pi\sqrt{(x / [kx/m])} = 2\pi\sqrt{(m/k)}$ $\qquad$ $T = 2\pi\sqrt{(m/k)}$

PERIOD OF A PENDULUM

Applying the SHM period relation to a pendulum whose length is L and whose bob mass is m, we find the following. Whenever the bob mass is at a distance x from equilibrium, the restoring force is mgx/L. (Trust me on this: ICBS.) Its acceleration is therefore $a = F/m = mgx/L\ m = gx/L$. So

$T = 2\pi\sqrt{(x/a)} = 2\pi\sqrt{(x / [gx/L])} = 2\pi\sqrt{(L/g)}$ $\qquad$ $T = 2\pi\sqrt{(L/g)}$

SO WHO USES THIS STUFF, ANYWAY

Anyone who needs to deal with harmonic motion of any kind must have a thorough understanding of these concepts and equations. One application is the harmonic damper used in some tall buildings. When the wind blows just wrong, some buildings start to oscillate like vertical diving boards. I'm sure you can appreciate the need to prevent this from happening. On the top floor of such buildings, a huge mass is driven back and forth (horizontally) to exactly counteract the effect of the wind. Jiggle the mass at the wrong period and, well, use your imagination...

