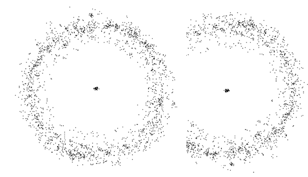
PhyzGuide: Hooke's Law

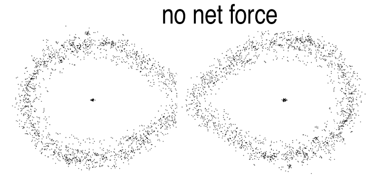
and the story of how things hang together

GOOD BONDS MAKE GOOD NEIGHBORS

In a solid, adjacent atoms or molecules are joined together by chemical bonds. The average distance between adjacent particles is referred to as their equilibrium distance. At this distance, the adjacent particles exert no net force on each other. If the particles are pulled apart so that their separation distance exceeds their equilibrium distance, they attract each other. If the particles are squeezed together, they repel.



EQUILIBRIUM
no net force



PULLED APART
each particle attracts the other

PUSHED TOGETHER
the particles repel

Ponder these questions: what fundamental force is responsible for the attraction or repulsion when the particles are not in equilibrium and what kind of equilibrium do the particles share?

This fundamental behavior can be seen on the large scale when we apply tension or compression forces on objects. When we exert a tension force on an object, it stretches; when we exert a compression force on an object, it compresses. (Springs and rubber bands show the effects under small loading forces, so we use them for demonstration and discussion purposes. But keep in mind that all solid objects show this behavior to some extent under some range of loading force.)

HOOKED ON CLASSICS

This characteristic of solid objects was stated by English physicist Robert Hooke in 1678.

The force of any spring is in the same proportion with its extension. That is, if one force stretch or bend it one space, two will bend it two, three will bend it three, and so forward.

Hooke was a contemporary of Newton's. The two were highly adversarial rivals. The mathematical expression of this statement is referred to as **Hooke's law**.

$$F = kx$$

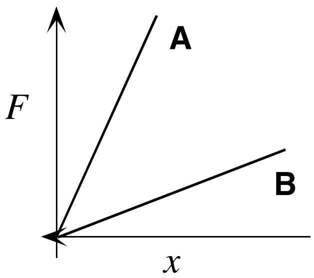
F is stressing force [N]
k is the force constant (spring constant) [N/m]
x is the distance of stretch or compression [m]

The force constant k is a measure of the stiffness of the *specific object* being stretched or compressed (there is no *single* force constant). The greater the value, the stiffer the object.

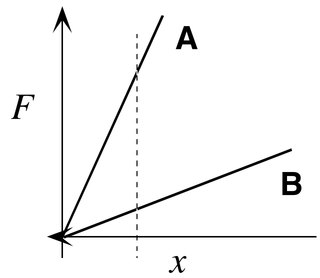
Hooke used his ideas to invent the spring scale. Spring scales are springs in housings calibrated to indicate forces. Calibration is simple, since a given force always causes the same stretch and more force always causes more stretch. Three centuries later, spring scales are still used in physics classes, supermarket produce sections and meat counters, and anywhere else an inexpensive way of determining a weight or other force is needed.

FORCE VS. STRETCH GRAPHS

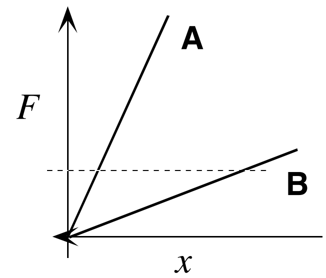
A common method used to determine force constants for objects is to load them with various weights, measure the corresponding stretches, and plot the results on a Force vs. Stretch graph.



Two springs, A and B were tested here. The slope of A's graph is greater than that of B's graph.



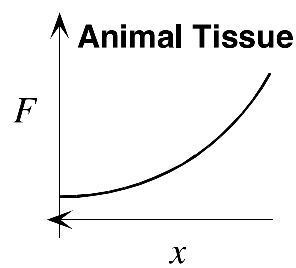
To get the same stretch, indicated by the vertical dotted line, more force must be applied to spring A.

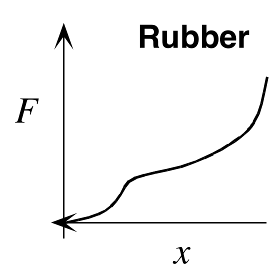


For the same stressing force, shown by the horizontal dotted line, spring B stretches more.

The force constant k is equal to the slope of the Force vs. Stretch graph. Stiffer springs yield graphs with greater slopes.

Any object whose Force vs. Stretch graph is a straight line is **hookean**. Rigid solids such as metals (from which we make springs) are hookean. Objects whose Force vs. Stretch graphs are curved are **non-hookean**. Rubber, polymers (plastics) and animal tissues are typically non-hookean.





Notice the animal tissue graph does not start at the origin; the tissue is under tension even when it is not stretched. Your skin is under tension like a tight Spandex® suit that has been stretched over your body.

ELASTIC AND PLASTIC

Whether or not an object is hookean, it will stretch to some extent when a stressing force acts on it. If it returns to its original shape or length when the load is removed, we call it **elastic**. If an object is permanently deformed after it is stretched, we say it is **plastic**. Note that we are not using the word plastic in its colloquial sense whereby it refers to polymer-based solids. Plastics (in the polymer sense) typically show elastic behavior under a certain range of stressing forces. Steel springs and rubber bands are elastic. Chewing gum and taffy are plastic.

Some objects that are elastic under certain stressing forces become plastic under higher stressing forces. When an object undergoes a transition from elastic to plastic behavior, we say that its **elastic limit** has been exceeded. The elastic limit is the stressing force at which the material passes from elastic to plastic behavior. Removing the load will not result in removal of the stretch; permanent deformation will have occurred.