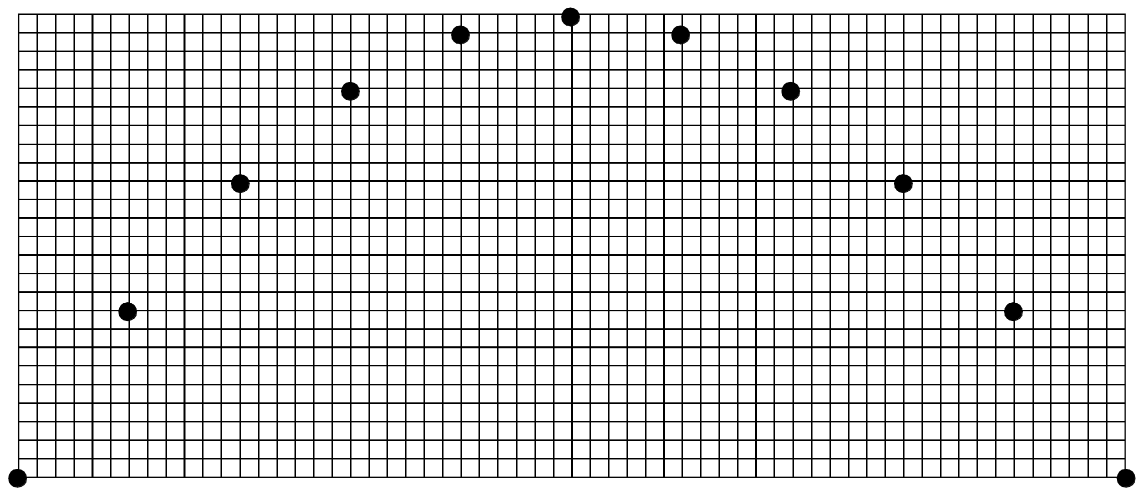
PhyzGuide: The Range Equation

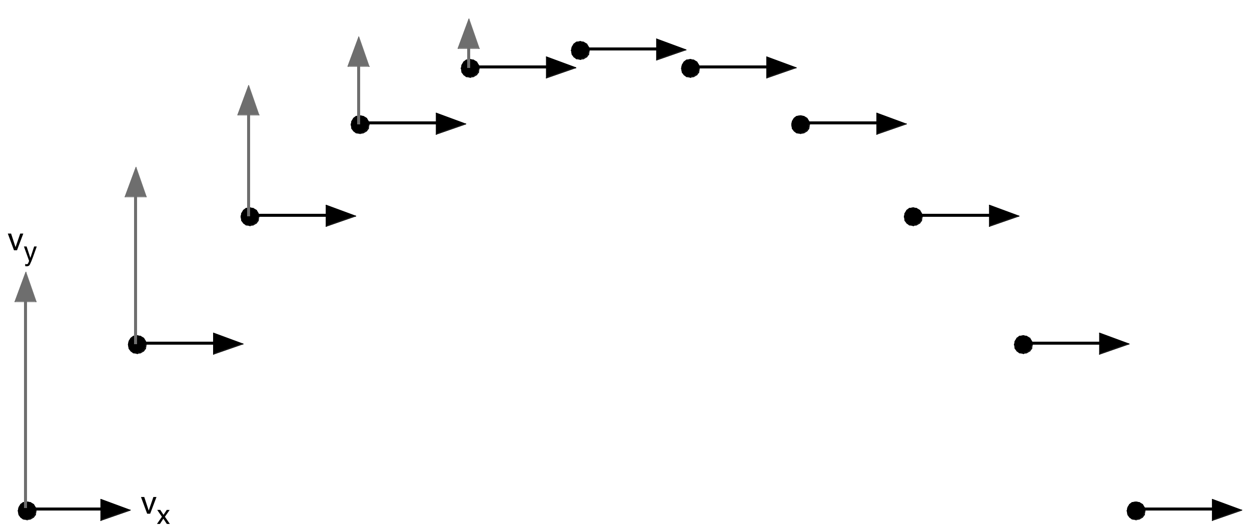
It is common for a projectile to be launched at an angle across a level surface. Military artillery shells (cannonballs), punted or passed footballs, batted or thrown baseballs, etc. We can plot and understand the nature of these projectiles if we remember to treat the horizontal and vertical components of the motion independently. Specifically, we treat the horizontal motion as uniform motion and the vertical motion as free fall (uniform accelerated motion).



Position. The illustration above shows a projectile launched at an angle across a level surface. It travels from left to right. We see the projectile at regular time intervals; if this were a photograph, it would involve multiple (11) exposures of the film taken in rapid succession.

The Horizontal Positions. Count the number of squares the projectile moves to the right between each exposure. Go ahead and write the number along the bottom of the illustration (a nice feature of *The Book of Phyz* is that you can write in it, highlight it, etc. without penalty). Does the number change as the flight progresses? Do you see why we say that the horizontal component of the motion is uniform motion?

The Vertical Positions. Count the number of squares the projectile moves upward between each exposure on the way up. Write the number along the left side of the illustration. Count the number of squares the projectile moves downward between each exposure on the way down. Write the number along the right side of the illustration. Does the number change as the flight progresses? Does it change in a consistent way (by about –2 squares between successive exposures)? Do you see why we say the vertical motion is uniform accelerated motion?



Velocity. The illustration above shows a projectile launched at an angle across a level surface. It travels from left to right. We see the projectile at regular time intervals; horizontal and vertical velocity component vectors have been added to the diagram.

The Horizontal Velocity. The projectile maintains uniform motion in the horizontal direction. The horizontal velocity $\mathbf{v_x}$ component vectors all have the same length. There is no horizontal acceleration of the projectile.

The Vertical Velocity. The projectile maintains uniform accelerated motion in the vertical direction. The vertical velocity $\mathbf{v_y}$ component vectors have different lengths. The vertical acceleration of the projectile is gravitational acceleration. Oh, I just noticed I forgot to continue the vertical velocity vectors past the apex (where $\mathbf{v_y} = 0$). Could you draw in the remaining five vectors in the illustration? Thanks. *Don't draw them in the wrong direction!*

Kinematics Equations. Suppose we knew the projectile's launch conditions: speed and angle. Could we then calculate the horizontal distance the projectile would cover before returning to the ground? Of course we can, we're physicists! Here's the general algebraic solution.

Given: v_0 and θ, find x.

x: UM	y: UAM
$x = ?$	$y = 0$ (we want to know x when the projectile returns to the ground)
$v_x = v_0\cos\theta$	$v_{0y} = v_0\sin\theta$
$t = ?$	$v_y = ?$
	$a = -g$ (negative because we chose a positive expression for v_{0y})

$x = v_x t$

$y = v_{0y}t + (1/2)at^2$

$0 = v_{0y}t + (1/2)at^2$

$0 = v_{0y} + (1/2)at$

$t = -2v_{0y} / a$

$x = v_x \cdot -2v_{0y} / a$

$x = v_0\cos\theta \cdot -2v_0\sin\theta / -g$

$x = (v_0^2/g) \cdot (2\cos\theta \sin\theta)$ Trig identity: $2\cos\theta\sin\theta = \sin 2\theta$

$x = (v_0^2/g) \sin 2\theta$

The Range Equation. The horizontal distance traveled by the projectile is often referred to as *range* and given the special symbol R. Therefore, the range equation is $R = (v_0^2/g) \sin 2\theta$.

With Numbers. For example, if our projectile were launched with a speed of 100 m/s at an angle of 60°, it would travel a distance of $(100\text{ m/s})^2 / 9.8\text{ m/s}^2 \cdot \sin 2\cdot 60° = 884$ m. Be careful that you use sin2θ and not 2sinθ in your calculation, and don't forget to square the initial speed.

Implications. According to the equation (and knowledge of the sine function), the ranges of projectiles launched with the same initial speed at complimentary angles are equal; the maximum range for a given initial speed is attained at a launch angle of 45°. Also, doubling the initial speed quadruples the range.

Caveat. Our treatment of the projectile neglects air resistance. Real projectiles encounter air resistance.