PhyzExamples: Advanced Kinematics

Physical Quantities • Symbols • Units • Brief Definitions

Displacement • x or y • m • The difference between where an object is now compared to where it was at a previous clock reading. "How far and in what direction." In each dimension of motion, the direction of displacement is either positive or negative. The decision as to which direction is positive and which is negative is arbitrary. (That means you can decide in each situation or problem, but you must be consistent within the situation or problem: Up and down cannot both be "positive.")

Interval • t • s • The time between two clock readings measured in finite units. Time progresses in only one "direction"; interval is always positive.

Velocity • v • m/s • The displacement through which an object moves in each unit of interval. (The rate at which position changes and the direction in which it changes.) "How fast and in what direction" In each dimension of motion, the direction of displacement is either positive or negative. The decision as to which direction is positive and which is negative is arbitrary.

> **Initial Velocity** • v_0 • The velocity of a body at the beginning of the interval for which its motion is considered. If a body "starts from rest," its initial velocity is zero.
>
> **Final Velocity** • v • The velocity of a body at the end of the interval for which its motion is considered. If the body "comes to a stop," its final velocity is zero.
>
> **Horizontal Velocity** • v_x • In projectile motion, a body has velocity in the horizontal direction and vertical direction. Motion in the horizontal direction is uniform motion. Thus v_{0x} and v_x are equal; use only v_x in problem-solving notation.
>
> **Vertical Velocity** • v_y • In projectile motion, a body has velocity in the horizontal direction and vertical direction. Motion in the vertical direction is uniform accelerated motion. Thus v_{0y} and v_y are unequal.

Acceleration • a • m/s^2 • The change in velocity an object experiences in each unit of interval. (The rate at which a body's velocity is changing.) In each dimension of motion, the direction of displacement is either positive or negative. The decision as to which direction is positive and which is negative is arbitrary.

> **Gravitational Acceleration** • g • 9.8 m/s^2 on Earth; other values on other worlds. **Free fall** is motion in one dimension with uniform acceleration caused by "gravity." Remember, you get to choose the direction that is positive. If you have a free fall or projectile motion problem, why not save yourself some trouble with negatives and choose "down" as positive?

Equations

$v = x/t$ • Uniform Motion • average velocity = displacement / interval

$v = v_0 + at$ • Uniform Accelerated Motion • final velocity = initial velocity + (acceleration · interval)

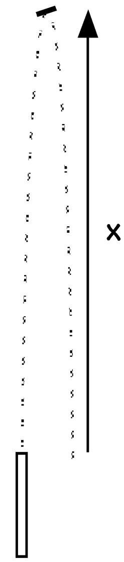
$x = (1/2)(v_0 + v)\cdot t$ • Uniform Accelerated Motion • displacement = (1/2) · (initial velocity + final velocity) · interval

$x = v_0t + (1/2)at^2$ • Uniform Accelerated Motion • displacement = initial velocity · interval + (1/2) · acceleration · square of the interval

$v^2 = v_0^2 + 2ax$ • Uniform Accelerated Motion • square of the final velocity = square of the initial velocity + 2 · acceleration · displacement

$x = vt - (1/2)at^2$ • Uniform Accelerated Motion • displacement = final velocity · interval – (1/2) · acceleration · square of the interval

Welcome to the Real World Examples



1. For entertainment one day, Mr. Baird blew a marker through an aluminum tube. He directed the trajectory straight upward. It took 2.4 s for the marker to reach its apex.

a. What was the initial velocity of the marker as it emerged from the tube?

1a. $x = ?$ $v_0 = ?$ $v = 0$ $a = -9.8\ m/s^2$ $t = 2.4\ s$

$v = v_0 + at$

$v_0 = v - at$

$v_0 = 0 - (-9.8\ m/s^2)(2.4\ s)$

$\underline{v_0 = 24\ m/s}$

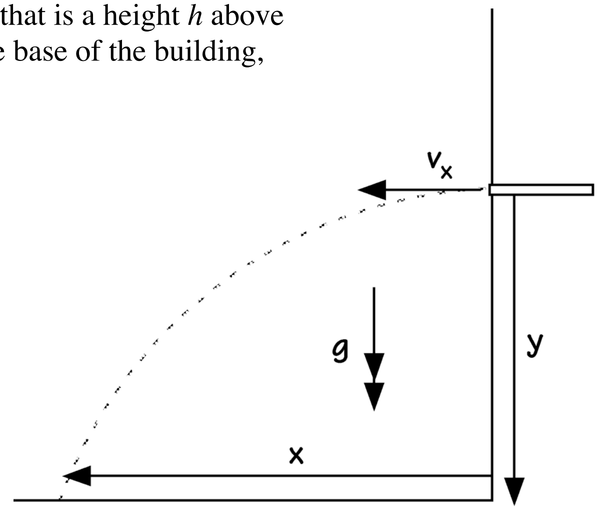
b. How high did the marker go?

$x = vt - (1/2)at^2$

$x = -(1/2)(-9.8\ m/s^2)(2.4\ s)^2$

$\underline{x = 28\ m}$

2. Next, he blew a marker horizontally out of a window that is a height *h* above the level ground below. If it landed a distance *d* from the base of the building, what was the initial (horizontal) speed of the marker?



2. x: UM y: UAM

$x = d$ $v_x = ?$ $t = ?$ $y = h$ $v_{y0} = 0$ $v_y = ?$ $a = g$

$v_x = x/t$

(but I don't know t, so...)

$y = v_{y0}t + (1/2)at^2$

$y = (1/2)at^2$

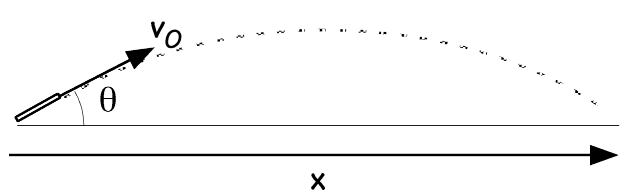
$t = \sqrt{(2y/a)}$

$t = \sqrt{(2h/g)}$

$v_x = d/\sqrt{(2h/g)}$

$\underline{v_x = d\sqrt{(g/2h)}}$

3. Next, he blew an identical marker through the tube at an angle of 30° above the horizontal. Assuming an initial speed of 24 m/s, and assuming the marker landed 132 m away, which planet was he on? ("Which planet?": Solve for *g*.)



3. x: UM

$x = 132\ m$

$v_x = v_0 \cdot \cos 30°$

$= 24\ m/s \cdot \cos 30$

$v_x = 21\ m/s$

$t = ?$

y: UAM

$y = ?$

$v_{y0} = v_0 \cdot \sin 30°$

$= 24\ m/s \cdot \sin 30°$

$v_{y0} = 12\ m/s$

$vy = -12\ m/s$

$a = ?$

$t = ?$

Use x to find t

$v_x = x/t$

$t = x/v_x$

Use y to find a

$v = v_0 + at$

$a = (v - v_0)/t$

$a = (-12\ m/s - 12\ m/s)\ /\ (132\ m\ /\ 21\ m/s)$

$a = -3.8\ m/s^2$

Planet: Mars!