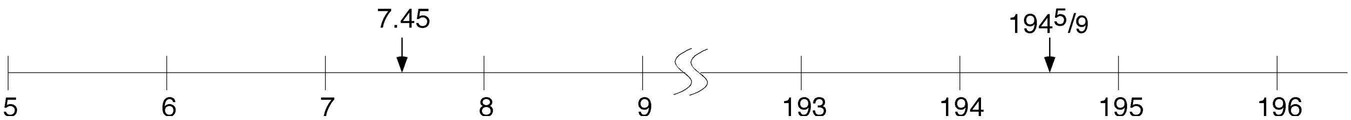
PhyzGuide: Orders of Magnitude

a new way of rounding

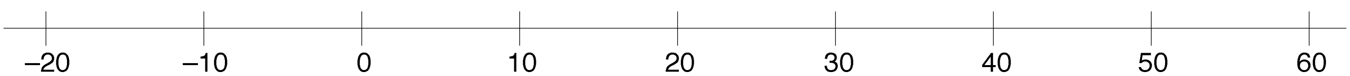
Despite what you might have heard elsewhere, it is not always important that the numbers used in scientific work always be precise. There are times when just knowing a “ball park” figure does quite nicely. This is true in estimation, especially when the numbers involved are large.

ROUNDABOUT

When rounding to the nearest “ones,” you are considering a number and determining which whole number it is nearest to. For example, 7.45 is nearest to 7, and $194^{5}/_{9}$ is nearest to 195. You can visualize this process by imagining a number line marked at every whole number. Every decimal or fractional number is closer to one whole number than it is to any another, except for those that end in .5 (or $^{1}/_{2}$).

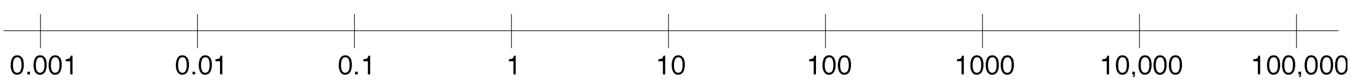


When you round to the nearest ten, you simply visualize a number line marked at every multiple of ten (i.e., 10, 20, 30, etc.). It is possible to round to the nearest multiple of any number using this process.

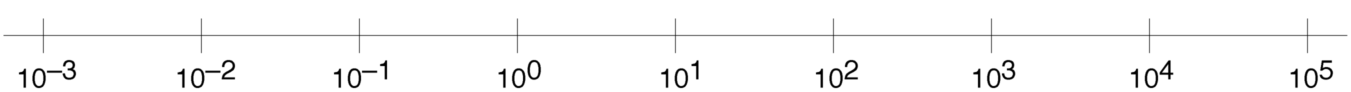


ROUNDING TO THE NEAREST POWER OF TEN

Now imagine rounding to the nearest *power* of ten. Visualize a number line with marks at 1, 10, 100, 1000, etc. It is also marked at 0.1, 0.01, 0.001, etc. Numbers can now be rounded by determining which power of ten they are nearest to. A number like 38 is nearer to 10 than it is to 100; 4,326.951 is nearer to 1,000 than is is to 10,000.



(Note that the number line is not to scale.) Suppose our new number line were marked in “power of ten” notation. The 1 would be labeled as 10^0, the 1000 as 10^3, the 0.01 as 10^{-2}, and so on. We could then identify rounded numbers by their corresponding power of ten. So 38 has an order of magnitude of 10^1; 4,326.951 has an order of magnitude 10^3.



THE SHARP CORNERS ON ORDER OF MAGNITUDE ROUNDING

The number 30 is nearer to 10 than it is to 100, so it rounds to 10^1; 70, however is nearer to 100 than it is to 10, so it rounds to 10^2. Can you determine the dividing line between rounding to 10^1 and rounding to 10^2?

RECAP

When one deals with numbers that differ in their powers of 10, it's helpful to think in terms of "orders of magnitude"—a fancy term for powers of 10. **Order of magnitude** refers only to the nearest power of 10 and not to the numerical coefficient. So the order of magnitude of 354,000 is 10^5. The order of magnitude of 754,000 is 10^6. The order of magnitude of 0.04 is 10^{-2}, while the order of magnitude of 0.06 is 10^{-1}.

PRACTICAL EXAMPLE: You're having lunch with friends and you think you have $10 but it turns out you only have $1. You tell your friends, "Wow, I overestimated my financial standing by an *ORDER OF MAGNITUDE*." You'll all enjoy a hearty chuckle and your friends will gladly cover you.

To the right, you will find order of magnitude listings for various lengths, masses, and time intervals. The numbers indicated on the bar are powers of ten, so 15 means 10^{15}. Bask in the trivia!